10/2/25 Sem - IV MJC - 06 Unit :- 01

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Potential Formulations of Electrodynamics

$$B = \text{curl } A \quad \text{--- (1)}$$

A is called the vector potential

After making this substitution equn can be re-written as

$$\text{curl}\left(E + \frac{\partial A}{\partial t}\right) = 0 \quad \text{--- (2)}$$

Since any quantity with a vanishing curl can be written as the gradient of a scalar.

$$E + \frac{\partial A}{\partial t} = -\nabla \phi$$

$$E = -\nabla \phi - \frac{\partial A}{\partial t} \quad \text{--- (3)}$$

where ϕ is the scaler potential. In electrostatics the potential ϕ alone determines E, but this is possible only under static conditions when E is a conservative field. To represent non-conservative E the additional term $\frac{\partial A}{\partial t}$ is required.

Substituting equn (3) in Maxwell's first equn.

$$\nabla^2 \phi + \frac{\partial}{\partial t}(\nabla \cdot A) = -\frac{\rho}{\epsilon_0} \quad \text{--- (4)}$$

The Maxwell's fourth eqⁿ changes as

$$\nabla \times (\nabla \times A) = \mu_0 J - \mu_0 \epsilon_0 \nabla\left(\frac{\partial \phi}{\partial t}\right) - \mu_0 \epsilon_0 \frac{\partial^2 A}{\partial t^2} \quad \text{--- (5)}$$

Using the vector identity

$$\nabla \times (\nabla \times A) = \nabla(\nabla \cdot A) - \nabla^2 A$$

and rearranging the terms.

$$\left(\nabla^2 A - \mu_0 \epsilon_0 \frac{\partial^2 A}{\partial t^2}\right) - \nabla\left(\nabla \cdot A + \mu_0 \epsilon_0 \frac{\partial \phi}{\partial t}\right) = -\mu_0 J \quad \text{--- (6)}$$

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